

Simulation of Partially Obscured Scenes

Using the Radiosity Method

by

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Introduction

Infrared (IR) Sensor Degradation due to :

- atmospheric emission, scattering and attenuation
- smoke
- fog
- clouds

Modelling Methods :

- radiative transfer (RT) methods :
 - discrete ordinates method (DOM)
 - Monte-Carlo method
- radiosity method

Capabilities of the Radiosity Method

- very complex scene geometries possible (e.g. inhomogeneous 3-D surface with inhomogeneous participating medium above)
- compute the radiative heat exchange between surfaces and participating medium
- Images of scenes can be rendered for any view-point, view-direction and field-of-view

Extended Radiosity Method

The extended radiosity method for illuminated surfaces A_i and volume elements V_k is based on the two coupled linear systems of equations for monochromatic radiation.

The energy balance equation for the i -th surface patch :

$$B_i^s A_i = E_i^s A_i + \rho_i \left[\sum_{j=1}^{N_s} B_j^s \frac{S_j S_i}{r_{ji}^2} + \sum_{k=1}^{N_v} B_k^v \frac{V_k S_i}{r_{ki}^2} \right],$$

$$i = 1, \dots, N_s, \quad (1)$$

and for the k -th volume element :

$$4 \kappa_{t,k} B_k^v V_k = 4 \kappa_{a,k} E_k^v V_k + \alpha_k \left[\sum_{j=1}^{N_s} B_j^s \frac{S_j V_k}{r_{jk}^2} + \sum_{m=1}^{N_v} B_m^v \frac{V_m V_k}{r_{mk}^2} \right],$$

$$k = 1, \dots, N_v, \quad (2)$$

where :

B_i^s = the radiosity in $[W \ m^{-2}]$,

$E_i^s A_i$ = is the emitted energy from surface i ,

ρ_i = is the reflectance of surface patch i ,

A_i = is the area of patch i ,

$4 \kappa_{t,k} B_k^v V_k =$ flux density leaving a volume element k ,

$B_k^v =$ is the volume radiosity in $[W \ m^{-3}]$,

$4 \kappa_{t,a} E_k^v V_k =$ is the emitted energy from volume k ,

$\kappa_{t,k} = \kappa_{a,k} + \kappa_{s,k}$,

$\kappa_{a,k} =$ is the absorption coefficient,

$\kappa_{s,k} =$ is the scattering coefficient,

$\alpha_k = \kappa_{s,k} / \kappa_{t,k} =$ is the scattering albedo,

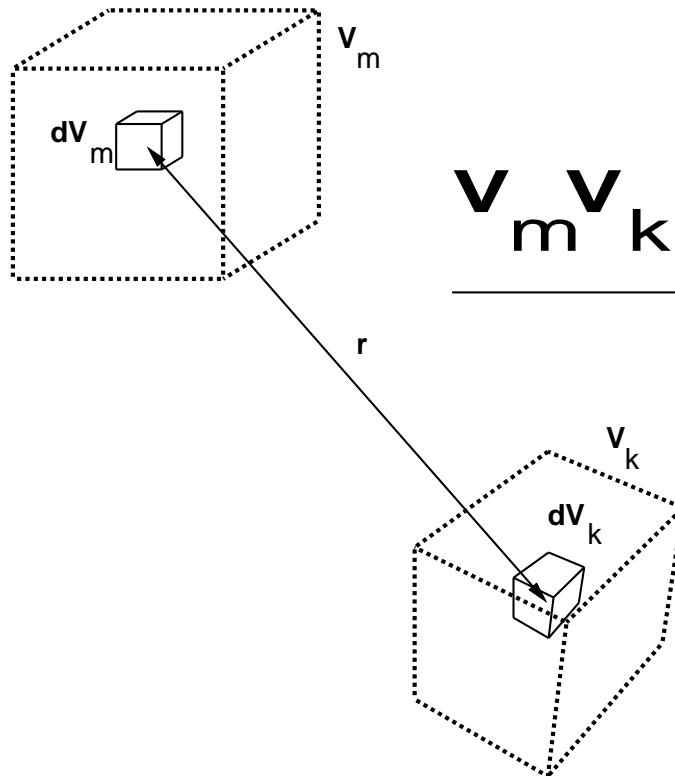
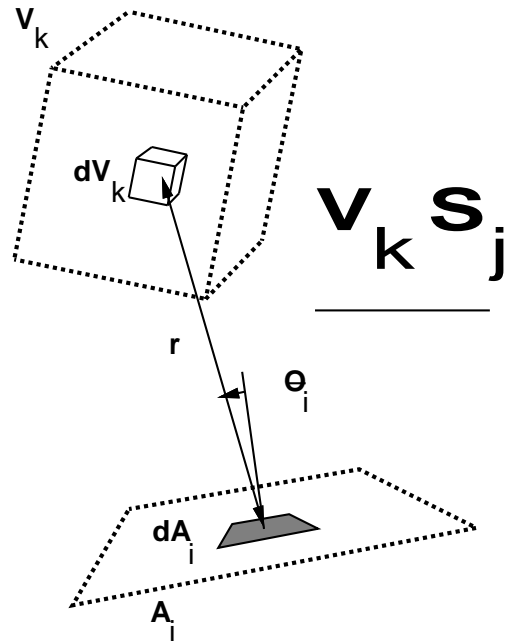
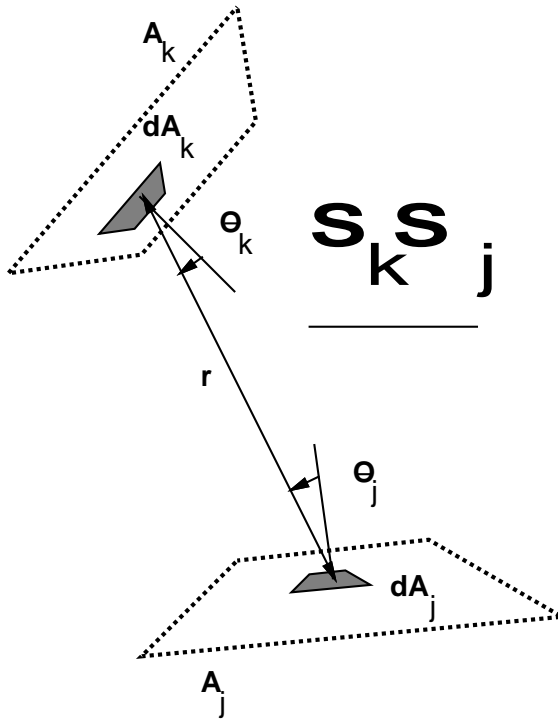
$\underline{S_k S_j} =$ is the view factor from a surface k to surface j ,

$\underline{V_k S_i} =$ is the view factor from volume k to surface i ,

$\underline{S_j V_k} =$ is the view factor from surface j to volume k ,

$\underline{V_m V_k} =$ is the view factor from volume m to volume k .

View Factors



View Factors

Definition : Fraction of energy reaching a volume or surface from another volume or surface

The view factor from surface k to surface j is defined as :

$$\underline{S_k S_j} = \int_{A_k} \int_{A_j} \frac{dA_k \cos \theta_k dA_j \cos \theta_j \tau(r)}{\pi r^2}. \quad (3)$$

The view factor from volume k to surface i is defined as :

$$\underline{V_k S_i} = \int_{V_k} \int_{A_i} \frac{\kappa_{t,k} dV_k dA_i \cos \theta_i \tau(r)}{\pi r^2}. \quad (4)$$

The view factor from volume m to volume k is defined as :

$$\underline{V_m V_k} = \int_{V_k} \int_{V_m} \frac{\kappa_{t,m} dV_m \kappa_{t,k} dV_k \tau(r)}{\pi r^2}. \quad (5)$$

The transmittance τ is given by :

$$\tau(r) = \exp \left[- \int_0^r \kappa_t(\chi) d\chi \right] \quad (6)$$

for a medium with variable κ_t along the line-of-sight path of length r .

Solution of the Radiosity Equations

The solution of eqs. (1,2) using eqs. (3)-(6) can be obtained using the Gauss-Seidel iteration scheme. The following two eqs. show the mechanism :

$$\begin{aligned}
 B_i^{s,l+1} = & E_i^s + \frac{\rho_i}{A_i} \sum_{k=1}^{N_v} B_k^{v,n} \frac{V_k S_i}{S_i} \\
 & - \frac{\rho_i}{A_i} \left[\sum_{j=1}^{i-1} B_j^{s,l+1} \frac{S_j S_i}{S_i} + \sum_{j=i+1}^{N_s} B_j^{s,l} \frac{S_j S_i}{S_i} \right], \\
 & i = 1, \dots, N_s
 \end{aligned} \tag{7}$$

and

$$\begin{aligned}
 B_k^{v,n+1} = & \frac{\kappa_{a,k}}{\kappa_{t,k}} E_k^v + \frac{\alpha_k}{4 \kappa_{t,k} V_k} \sum_{j=1}^{N_s} B_j^{s,l+1} \frac{S_j V_k}{V_k} \\
 & - \frac{\alpha_k}{4 \kappa_{t,k} V_k} \left[\sum_{j=1}^{k-1} B_j^{v,n+1} \frac{V_j V_k}{V_k} \right. \\
 & \left. + \sum_{j=k+1}^{N_v} B_j^{v,n} \frac{V_j V_k}{V_k} \right], \\
 & k = 1, \dots, N_v
 \end{aligned} \tag{8}$$

where superscripts l and n denote the iteration.

Criterion to stop iteration:

$$| B_i^{s,l+1} - B_i^{s,l} | < \varepsilon \text{ for all } i = 1, \dots, N_s \quad (9)$$

and

$$| B_k^{v,n+1} - B_k^{v,n} | < \varepsilon \text{ for all } k = 1, \dots, N_v. \quad (10)$$

Number of required iterations : 10 to 30 for $\varepsilon = 10^{-9}$

Rendering of Radiosity Solutions

Compute radiance I [$W\ m^{-2}\ sr^{-1}$] :

$$I(L) = \tau(L) \frac{B_i^s}{\pi} + \int_0^L \tau(l) \frac{B^v(l)}{\pi} \kappa_t(l) dl, \quad (11)$$

where :

$I(L)$ = is the radiance at the observer location L ,

B_i^s = is the surface radiosity,

$B^v(l)$ = is the volume radiosity along the line-of sight,

Practical implementation :

$$I(L) = \exp(-\kappa_t L) \frac{B_i^s}{\pi} + \sum_{k=1}^K \exp(-\kappa_t k \Delta l) \frac{B_k^v}{\pi} \kappa_t \Delta l, \quad (12)$$

where $\Delta l = \frac{L}{K \cos \theta_r}$

Example of a Scene Simulation Using the Extended Radiosity Method

Assumptions :

- A parallelepiped of a homogeneous participating medium is located above a flat Lambertian surface.
- The surface consists of $N_s = N_x \times N_y$ rectangular patches with varying reflectance.
- The parallelepiped is divided into $N_v = N_x \times N_y \times N_z$ volume elements.
- The illumination source is a point source at infinity with illuminating rays from the direction (θ_s, ϕ_s) .
- The observer is located at (x_0, y_0, z_0) .

The energy balance equations for the (i_x, i_y) -th surface patch :

$$\begin{aligned}
 B_{i_x, i_y}^s dA &= E_{i_x, i_y}^s dA \\
 &+ \rho_{i_x, i_y} \sum_{k_x=1}^{N_x} \sum_{k_y=1}^{N_y} \sum_{k_z=1}^{N_z} B_{k_x, k_y, k_z}^v \frac{V_{k_x, k_y, k_z} S_{i_x, i_y}}{i_x = 1, \dots, N_x; i_y = 1, \dots, N_y}
 \end{aligned} \quad (13)$$

where $dA = \Delta x \Delta y$ and for the (k_x, k_y, k_z) -th volume element :

$$\begin{aligned}
 4 \kappa_t B_{k_x, k_y, k_z}^v dV &= 4 \kappa_t E_{k_x, k_y, k_z}^v dV \\
 &+ \alpha \left[\sum_{j_x=1}^{N_x} \sum_{j_y=1}^{N_y} B_{j_x, j_y}^s \frac{S_{j_x, j_y} V_{k_x, k_y, k_z}}{j_x = 1, \dots, N_x; j_y = 1, \dots, N_y} \right. \\
 &+ \sum_{m_x=1}^{N_x} \sum_{m_y=1}^{N_y} \sum_{m_z=1}^{N_z} B_{m_x, m_y, m_z}^v \\
 &\left. \frac{V_{m_x, m_y, m_z} V_{k_x, k_y, k_z}}{m_x = 1, \dots, N_x; m_y = 1, \dots, N_y; m_z = 1, \dots, N_z} \right],
 \end{aligned} \quad (14)$$

where $dV = \Delta x \Delta y \Delta z$.

The volume/surface and the surface/volume view factors are approximately given by :

$$\frac{V_{k_x, k_y, k_z} S_{i_x, i_y}}{S_{i_x, i_y} V_{k_x, k_y, k_z}} = \frac{S_{i_x, i_y} V_{k_x, k_y, k_z}}{\kappa_t dV^2 k_z^2 \tau(k_x, k_y, k_z; i_x, i_y)} \approx \frac{\tau(k_x, k_y, k_z; i_x, i_y)}{\pi \{r(k_x, k_y, k_z; i_x, i_y)\}^3} \quad (15)$$

where

$$r(k_x, k_y, k_z; i_x, i_y) =$$

$$\sqrt{[(k_x - i_x)\Delta x]^2 + [(k_y - i_y)\Delta y]^2 + [k_z - i_z]^2}$$

and with transmittance :

$$\tau(k_x, k_y, k_z; i_x, i_y) = \exp[-\kappa_t r(k_x, k_y, k_z; i_x, i_y)]. \quad (16)$$

The volume/volume view factors according to eq. (5) are approximately :

$$\frac{V_{m_x, m_y, m_z} V_{k_x, k_y, k_z}}{V_{k_x, k_y, k_z} V_{m_x, m_y, m_z}} = \frac{V_{k_x, k_y, k_z} V_{m_x, m_y, m_z}}{\kappa_t^2 dV^2 \tau(m_x, m_y, m_z; k_x, k_y, k_z)} \approx \frac{\tau(m_x, m_y, m_z; k_x, k_y, k_z)}{\pi \{r(m_x, m_y, m_z; k_x, k_y, k_z)\}^3} \quad (17)$$

where

$$r(m_x, m_y, m_z; k_x, k_y, k_z) =$$

$$\sqrt{[(k_x - m_x)\Delta x]^2 + [(k_y - m_y)\Delta y]^2 + [(k_z - m_z)\Delta z]^2}$$

and with transmittance :

$$\tau(k_x, k_y, k_z; m_x, m_y, m_z) = \exp[\kappa_t r(m_x, m_y, m_z; k_x, k_y, k_z)]. \quad (18)$$

Symmetry Exists

e.g.

$$\underline{V_{5,6,10} S_{1,1}} = \underline{V_{6,7,10} S_{2,2}} = \underline{V_{10,11,10} S_{6,6}} = \dots \text{ etc}$$

and

$$\underline{V_{5,6,10} V_{1,1,1}} = \underline{V_{6,7,11} V_{2,2,2}} = \underline{V_{10,11,12} V_{6,6,3}} = \dots \text{ etc.}$$

Complexity of Solution :

Number of necessary multiplications in one iteration :

$$N_{mult} = (N_x N_y N_z)^2 + 2 N_x^2 N_y^2 N_z. \quad (19)$$

Approximation :

Approximative solution using radiosities in a parallelepiped of dimensions $M_x \Delta x \times M_y \Delta y \times M_z \Delta z$ centered around the volume element. Number of multiplications :

$$M_{mult} = N_x N_y N_z M_x M_y M_z + 2 N_x M_x N_y M_y N_z, \quad (20)$$

Computation Example

Hardware : SparcStation which is rated at 15.8 MIPS and 1.7 MFLOPS.

Problem : $N_x = N_y = 100$, $N_z = 10$ and $M_x = M_y = 10$, $M_z = 4$

CPU time : 22 minutes and 45 seconds per iteration

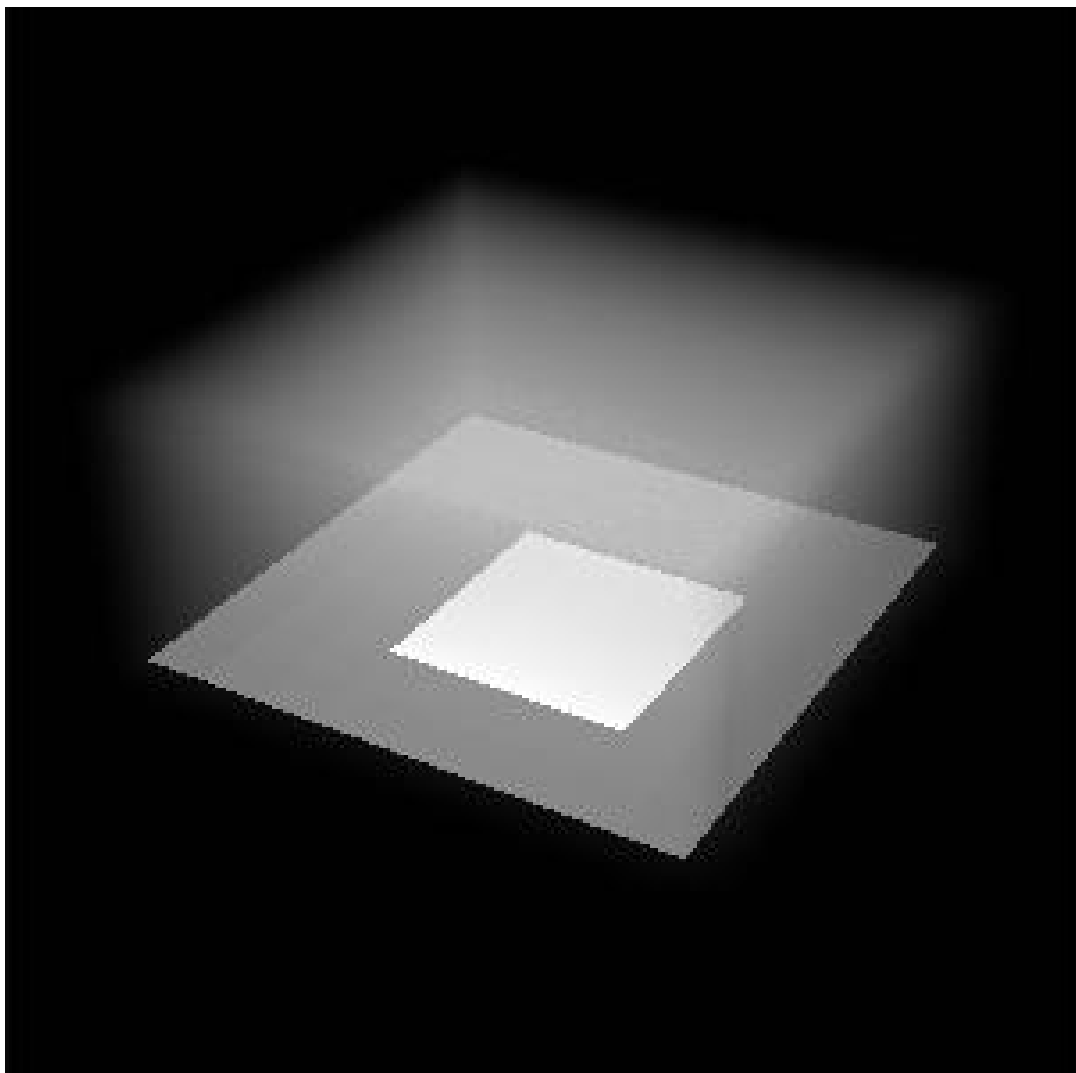
Image Generation Example

Input :

- The participating medium had an absorption coefficient of $\kappa_a = 0.3$ and a total scattering coefficient $\kappa_t = 0.8$.
- The dimensions of a volume element were $\Delta x = \Delta y = 0.1$ and $\Delta z = 0.05$ with $N_x = N_y = N_z = 10$.
- The surface had a reflectance of $\rho = 0.3$ with a higher reflectance of $\rho = 0.6$ in a square in the middle for $i_x = 4, 5, 6, 7$ and $i_y = 4, 5, 6, 7$.

Performance :

- The Gauss-Seidel iteration converged after 13 iterations with 73 CPU seconds.
- The rendering of the 300 by 300 image using 10 interpolated radiositities along the line of sight took 73 seconds to compute.



Synthetic image of a scene obscured
by a participating medium.

Conclusions

- The extended radiosity method is shown to be practicable and useful to generate realistic scenes taking into account multiple scattering between surfaces and a participating volumetric medium, such as smoke or another obscurant.
- The effects of such obscurants on surface images would otherwise be difficult or impossible to compute with either the standard radiosity methods or the 3-D radiative transfer methods required to treat such problems.